Instructions: Try these problems from the textbook as practice (answers below).

Problems

5-17. Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{S}$; that is, calculate the flux of $\mathbf{F}$ across S

7. $\mathbf{F}(x, y, z) = \langle 3xy^2, xe^z, z^3 \rangle$, S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$
9. $\mathbf{F}(x, y, z) = \langle xe^y, z - e^y, -xy \rangle$, S is the ellipsoid $x^2 + 2y^2 + 3z^2 = 4$
11. $\mathbf{F}(x, y, z) = \langle 2x^3 + y^3, y^3 + z^3, 3y^2z \rangle$, S is the surface of the solid bounded by the paraboloid $z = 1-x^2-y^2$ and the xy-plane
17. $\mathbf{F} = |\mathbf{r}|\mathbf{r}$, where $\mathbf{r} = \langle x, y, z \rangle$, S consists of the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and the disk $x^2 + y^2 \leq 1$ in the xy-plane

19. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle z^2x, y^3/3 + \tan^{-1} z, x^2z + y^2 \rangle$ and S is the top half of the sphere $x^2 + y^2 + z^2 = 1$.

Hint: Note that S is not a closed surface. First compute integrals over S_1 and S_2 where S_1 is the disk $x^2 + y^2 \leq 1$, oriented downward, and $S_1 = S \cup S_1$.

Answers

7. $9\pi/2$
9. 0
11. π
17. 2π
19. $13\pi/20$